a) Find a formula for $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\cdots+\frac{1}{n(n+1)}$ by examining the values of this expression for small values of n.

b) Prove the formula you conjectured in part (a).

Prove that if $h>-1$, then $1+nh\leq(1+h)^n$ for all nonnegative integers n.

This is called **Bernoulli's inequality.**

Prove that if n is a positive integer, then 133 divides $11^{n+1}+12^{2n-1}$.

Show that if A_1, $A_2, \ldots,$ A_n are sets where $n \geq 2$, and for all pairs of integers i and j with $1 \leq i < j \leq n$ either A_i is a subset of A_j or A_j is a subset of A_i, then there is an integer i, $1 \leq i \leq n$ such that A_i is a subset of A_j for all integers j with $1 \leq j \leq n$.

Use strong induction to prove that $\sqrt{2}$ is irrational. [*Hint:* Let $P(n)$ be the statement that $\sqrt{2} \neq n/b$ for any positive integer b]

Find the flaw with the following "proof" that $a^n = 1$ for all nonnegative integers n, whenever a is a nonzero real number.

Basis step: $a^0 = 1$ is true by the definition of a^0.

Inductive step: Assume that $a^j = 1$ for all nonnegative integers j with $j \leq k$. Then note that

$$a^{k+1} = \frac{a^k \cdot a^k}{a^{k-1}} = \frac{1 \cdot 1}{1} = 1.$$

Give a recursive definition of

a) the set of odd positive integers.

b) the set of positive integer powers of 3.

c) the set of polynomials with integer coefficient.

a) Give a recursive definition of the function *ones*(*s*), which counts the number of ones in a bit string *s*.

b) Use structural induction to prove that *ones*(*st*) = *ones*(*s*) + *ones*(*t*).

Recursively define the set of bit strings that have more zeros than ones.

Describe a recursive algorithm for multiplying two nonnegative integers x and y based on the fact that $xy = 2(x \cdot (y/2))$ when y is even and $xy = 2(x \cdot \lfloor y/2 \rfloor) + x$ when y is odd, together with the initial condition $xy = 0$ when $y = 0$.

Devise a recursive algorithm for computing n^2 where n is a nonnegative integer using the fact that $(n+1)^2 = n^2 + 2n + 1$. Then prove that this algorithm is correct.

Devise a recursive algorithm to find the nth term of the sequence defined by $a_0 = 1,\ a_1 = 2$, and $a_n = a_{n-1} \cdot a_{n-2}$, for $n = 2, 3, 4$ ...

Give a recursive algorithm for finding the string w^i, the concatenation of i copies of w, when w is a bit string.